

$$16a) \lambda^2 + \lambda - 12 = 0 \Rightarrow (\lambda - 3)(\lambda + 4) = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = 3, \lambda^{(B)} = -4 \quad Y = C^{(A)} e^{+3x} + C^{(B)} e^{-4x} \quad (4P)$$

$$b) 6\lambda^2 + 5\lambda - 6 = 0 \Rightarrow (3\lambda - 2)(2\lambda + 3) = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = \frac{2}{3}, \lambda^{(B)} = -\frac{3}{2} \quad Y = C^{(A)} e^{\frac{2}{3}x} + C^{(B)} e^{-\frac{3}{2}x} \quad (4P)$$

$$c) \lambda^2 + 2\lambda + 17 = 0 \Rightarrow (\lambda + 1)^2 + 16 = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = -1 + 4i, \lambda^{(B)} = -1 - 4i$$

$$\Rightarrow Y = C^{(A)} e^{(-1+4i)x} + C^{(B)} e^{(-1-4i)x} \quad (4P)$$

$$d) \lambda^2 + 2\lambda + 3 = 0 \Rightarrow (\lambda + 1)^2 + 2 = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = -1 + i\sqrt{2}, \lambda^{(B)} = -1 - i\sqrt{2}$$

$$\Rightarrow Y = C^{(A)} e^{(-1+i\sqrt{2})x} + C^{(B)} e^{(-1-i\sqrt{2})x} \quad (4P)$$

$$e) 16\lambda^2 + 8\lambda + 1 = 0 \Rightarrow (4\lambda + 1)^2 = 0 \quad (2P)$$

$$\lambda = -\frac{1}{4} \quad Y = C^{(A)} e^{-x/4} + C^{(B)} x e^{-x/4} \quad (4P)$$

$$17a) 10\lambda^2 - \lambda - 3 = (5\lambda - 3)(2\lambda + 1) = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = \frac{3}{5}, \lambda^{(B)} = -\frac{1}{2} \quad (2P)$$

$$Y = C^{(A)} e^{\frac{3}{5}x} + C^{(B)} e^{-x/2} \quad (2P)$$

$$Y' = C^{(A)} \frac{3}{5} e^{\frac{3}{5}x} - C^{(B)} \frac{1}{2} e^{-x/2}$$

$$1 = Y(0) = C^{(A)} + C^{(B)} \quad (2P)$$

$$0 = Y'(0) = \frac{3}{5}C^{(A)} - \frac{1}{2}C^{(B)}$$

$$\Rightarrow C^{(A)} = \frac{5}{11}, C^{(B)} = \frac{6}{11} \quad (2P)$$

$$b) \lambda^2 + 25 = 0 \Rightarrow \lambda^{(A)} = 5i, \lambda^{(B)} = -5i \quad (2P)$$

$$Y = C^{(A)} e^{5ix} + C^{(B)} e^{-5ix} \quad (2P)$$

$$Y' = C^{(A)} 5i e^{5ix} - C^{(B)} 5i e^{-5ix}$$

$$1 = Y(0) = C^{(A)} + C^{(B)} \quad (2P)$$

$$-1 = Y'(0) = 5i(C^{(A)} - C^{(B)})$$

$$\Rightarrow C^{(A)} = \frac{1}{2} + \frac{i}{10}, C^{(B)} = \frac{1}{2} - \frac{i}{10} \quad (2P)$$

$$c) \lambda^2 - 2\lambda - 3 = (\lambda + 1)(\lambda - 3) = 0 \quad (2P)$$

$$\lambda^{(A)} = -1, \lambda^{(B)} = 3 \quad (2P)$$

$$Y = C^{(A)} e^{-x} + C^{(B)} e^{3x} \quad (2P)$$

$$Y' = -C^{(A)} e^{-x} + 3C^{(B)} e^{3x}$$

$$2 = Y(0) = C^{(A)} + C^{(B)} \quad (2P)$$

$$-3 = Y'(0) = -C^{(A)} + 3C^{(B)}$$

$$\Rightarrow C^{(A)} = \frac{9}{4}, C^{(B)} = -\frac{1}{4} \quad (2P)$$

$$d) \lambda^2 - 4\lambda - 5 = (\lambda + 1)(\lambda - 5) = 0 \quad (2P)$$

$$\lambda^{(A)} = -1, \lambda^{(B)} = 5$$

(2P)

$$y = C^{(A)} e^{-x} + C^{(B)} e^{5x}$$

(2P)

$$y' = -C^{(A)} e^{-x} + 5C^{(B)} e^{5x}$$

$$-1 = y'(0) = C^{(A)} + 5C^{(B)}$$

(2P)

$$C^{(A)} = -2/3$$

(2P)

$$C^{(B)} = -1/3$$

$$e) \lambda^2 - 4\lambda + 13 = (\lambda - 2)^2 + 9 = 0$$

(2P)

$$\lambda^{(A)} = 2 + 3i, \lambda^{(B)} = 2 - 3i$$

(2P)

$$y = C^{(A)} e^{(2+3i)x} + C^{(B)} e^{(2-3i)x}$$

(2P)

$$y' = (2+3i)C^{(A)} e^{(2+3i)x} + C^{(B)}(2-3i)e^{(2-3i)x}$$

$$4 = y(0) = C^{(A)} + C^{(B)}$$

(2P)

$$0 = y'(0) = (2+3i)C^{(A)} + (2-3i)C^{(B)} = 8 + 3i(C^{(A)} - C^{(B)})$$

$$\Rightarrow C^{(A)} = 2 + \frac{4}{3}i, C^{(B)} = 2 - \frac{4}{3}i$$

(2P)

$$18) \text{ A and IV) } \quad (1P)$$

$$(x^2_1 \lambda^2 + \lambda + 1 = \frac{1}{4}(\lambda + 2)^2 = 0 \Rightarrow \lambda = -2 \text{ repeated})$$

$$\text{B and V) } \quad (1P)$$

$$(2(\lambda^2 + \lambda + \frac{1}{2}) = 2((\lambda + \frac{1}{2})^2 + \frac{1}{4}) = 0 \Rightarrow \lambda = -\frac{1}{2} \pm i\frac{1}{2})$$

$$\text{C and II) } \quad (1P)$$

$$(2(\lambda^2 - 4\lambda + 4) = 2(\lambda - 2)^2 = 0 \Rightarrow \lambda = 2 \text{ repeated})$$

$$\text{D and I) } \quad (1P)$$

$$(\lambda^2 + \lambda - 2 = (\lambda + 2)(\lambda - 1) = 0 \Rightarrow \lambda^{(A)} = -2, \lambda^{(B)} = 1)$$

$$\text{E) and III) } \quad (1P)$$

$$(\lambda^2 - 2\lambda + 3 = (\lambda - 1)^2 + 2 = 0 \Rightarrow \lambda^{(A/B)} = 1 \pm i\sqrt{2})$$

$$\text{F) } \quad (10P)$$

$$z' = -\frac{y''}{x-1} + \frac{y'}{(x-1)^2} (x-1' + y)$$

$$= -\frac{y' - x^3 y}{x^2 y} + \frac{y'(x y' + y)}{x^2 y^2}$$

$$(\text{using } xy'' = y' - x^3 y)$$

$$\equiv x + \frac{y'^2}{x y^2} = x + x z^2$$

(10P)

solution by separation

("Riccati equation")

$$\frac{dz}{1+z^2} = x dx \Rightarrow \arctan(z) = \frac{1}{2} x^2 + C_1$$

$$\Rightarrow z = \tan(\frac{1}{2} x^2 + C_1)$$

(5P)

$$y' = -z \cdot xy = -\tan(\frac{1}{2} x^2 + C) \cdot x \cdot y$$

solution by separation

$$\frac{dy}{y} = -\tan\left(\frac{1}{2}x^2 + C\right) \cdot x$$

$$\Rightarrow \ln|y| = + \ln|\cos\left(\frac{1}{2}x^2 + C\right)| + D$$

$$\Rightarrow y = e^D \cdot \cos\left(\frac{1}{2}x^2 + C\right)$$

(5P)

$$(= A \cos\left(\frac{1}{2}x^2\right) + B \sin\left(\frac{1}{2}x^2\right))$$